1. B $(x+1)^2+(y-3)^2=10 \rightarrow A=10, h=-1, k=3 \rightarrow Ah+k=-7$

2. B The point (1,2) is on the first line. Thus, using the equation for the distance from a point to the second line gives $\frac{|1+4+5|}{\sqrt{1^2+2^2}}=\frac{10}{\sqrt{5}}$

3. D Simply $\pi r^2=2018^2\pi$

4. D $\frac{(x-1)^2}{3}+\frac{(y-8)^2}{9}=1 \rightarrow a^2=9, b^2=3, c=\sqrt{a^2-b^2}=\sqrt{6}$. The center is at (1,8) and the directrices occur at $\frac{a^2}{c}$ to the top and bottom of the ellipse, thus occuring at $y=8-\frac{9}{\sqrt{6}}, 8+\frac{9}{\sqrt{6}}$. Thus, $p-q$ yields $\frac{18}{\sqrt{6}}$.

5. D Just a parabola so answer is 1.

6. E Answer is NOT a hyperbola because the distance of the foci is $\sqrt{2}$. The right side of the equation is a number larger than $\sqrt{2}$ meaning that this curve does not exist.

7. B $2i+3j-7k=0$. Plug in v for i, t for k to get that $j=\frac{7t-2v}{3}$. Multiply all by 3 to get $\langle 3v, 7t-2v, 3t\rangle$.

8. C I is true from $(x-1)(x+1)=0, (x-1)^2=0, x^2=-1$. II is true from $x^2+y^2=0, x^2+y^2=-1$, III is true because it is the asymptotes. IV. is false from $xy=1$.

9. D There are $\frac{n}{2}$ chords from the points that are diameters. From each chord, the third point will always form a right triangle due to the 180° subtended arc so you have $n-2$ points to choose from. Thus, the answer is $\frac{\frac{n}{2}*(n-2)}{\binom{n}{3}}=\frac{3}{n-1}$.

10. D Instead of trying to find the number of acute triangles, let's find the number of obtuse triangles. There are 1008 sets of 2018 chords with equal lengths that are not the diameter. For any chosen chord, all points in the smaller subtended arc of the chord will form an obtuse triangle with the chord. Thus, the number of obtuse triangles is equal to $2018\sum_{n=0}^{1007} n$. Thus, the probability to make an obtuse triangle is $\frac{2018*1008*1007}{2*\binom{2018}{3}}=\frac{3021}{4034}$. From question 9, the probability to make a right triangle is $\frac{3}{2017}$. Thus, the probability to make an acute triangle is $1-\frac{3021}{4034}-\frac{3}{2017}=\frac{1007}{4034}$. $b-a=3027$

11. C $e=\frac{f}{d}$ where f is the distance from a point on the conic to the focus while d is the distance from that point to the directrix. $f=1, d=\frac{|3-1|}{\sqrt{3^2+4^2}} \rightarrow \frac{f}{d}=\frac{5}{2}$.

12. B Let (x,y) represent the points on the conic. Thus, those points must meet the criteria for the eccentricity so $f=\sqrt{(x-1)^2+(y-1)^2}, d=\frac{|3x-4y-1|}{5} \rightarrow \frac{f}{d}=\frac{5}{2} \rightarrow 4f^2=25d^2 \rightarrow 8-8x+4x^2-8y+4y^2=(-1+3x-4y)^2=1-6x+9x^2+8y-24xy+16y^2 \rightarrow 1-6x+9x^2+8y-24xy+16y^2-(8-8x+4x^2-8y+4y^2)=0$ so simplifying gives $-7+2x+5x^2+16y-24xy+12y^2=0$. The sum is 11.

13. E These lines are parallel so the y-intercepts are opposites of each other. Thus, the difference of the slopes and the sum of the negative y-intercepts are both 0. Thus, the answer is 0.

14. A $r = 8\cos^4\left(\frac{\theta}{4}\right) - 4\cos\left(\frac{\theta}{2}\right) = 8\left(\frac{1+\cos\left(\frac{\theta}{2}\right)}{2}\right)^2 - 4\cos\left(\frac{\theta}{2}\right) = 8\left(\frac{1+2\cos\left(\frac{\theta}{2}\right)+\cos^2\left(\frac{\theta}{2}\right)}{4}\right) - 4\cos\left(\frac{\theta}{2}\right) = 2\left(1+2\cos\left(\frac{\theta}{2}\right)+\frac{1+\cos(\theta)}{2}\right) - 4\cos\left(\frac{\theta}{2}\right) = 3+\cos(\theta)$ →Convex limacon because $a = 3b$

15. B $r = \sin(\theta)\cos(\theta)\cos(2\theta)\cos(4\theta)\ldots\cos(2^n\theta) = \frac{\sin(2^{n+1}\theta)}{2^{n+1}}$. Area is given by $\frac{a^2\pi}{2} = \frac{\pi}{2^{2n+2+1}}$. Number of petals is 2*$2^{n+1} = 2^{n+2}$. Product is $\frac{1}{2^{n+1}}$

16. A $a = 2, b = 3,$ latus rectum $= \frac{2b^2}{a} = 9$

17. B Sketching the region, we see that it is a triangle below the y-axis. Start by choosing a x_1 value on the line $y = -3x + 2$. The corresponding x value for the second line is solved through $-3x_1 + 2 = x - 3$, yielding $x = -3x_1 + 5$. Thus, the length of our rectangle is $-3x_1 + 5 - x_1 = -4x_1 + 5$, and the height of our rectangle is $-(-3x_1 + 2)$, because the y value is negative. The area is $-(-3x_1 + 2)(-4x_1 + 5) = -12x^2 + 23x - 10$. This is a parabola, which has its vertex at $-\frac{b}{2a} = \frac{23}{24}$. Plugging that in gives us $\frac{49}{48}$ as the maximum.

18. B II,III,IV,VI

19. D $\sinh(t) = \frac{(x-2)}{3}, \cosh(t) = \frac{(y-3)}{4} \rightarrow \cosh^2(t) - \sinh^2(t) = 1 = \frac{(y-3)^2}{16} - \frac{(x-2)^2}{9}$. center $= (2,3), a = 3, b = 4 \rightarrow$ slope $= \pm\frac{b}{a} = \pm\frac{4}{3}$. Thus, the two possible asymptotes occur at $y - 3 = \pm\frac{4}{3}(x-2) \rightarrow \pm\frac{3}{4}(y-3) = x - 2 \rightarrow \frac{3}{4}y - \frac{9}{4} = x - 2$ or $-\frac{3}{4}y + \frac{9}{4} = x - 2$. Choosing the latter gives $x + \frac{3}{4}y - \frac{17}{4} = 0$

20. C Subtract each point by the origin to make 2 vectors $(1,2,3)$ and $(e, e, 1)$. Take the cross product to get $\langle 2-3e, 3e-1, -e\rangle$. The area is half of the magnitude: $\frac{\sqrt{5-18e+19e^2}}{2}$

21. B Referring to the expansion given at the front of the test, we have $f + ah^2 + bhk + ck^2 - 2ahx - bkx + ax^2 - bhy - 2cky + bxy + cy^2 = x^2 + xy + y^2 + 2x - 8y + 4$. We use f instead of 1 to account for multiplying the conic equation by a nonzero constant. This gives us the equations" $a = 1, c = 1, -2ah - bk = 2, -bh - 2ck = -8$, $-f + ah^2 + bhk + ck^2 = 4$. Solving gives $a = 1, b = 1, c = 1, h = -4, k = 6, f = 24$. The center is $(h,k) = (-4,6)$. We see that the line contains the center of the ellipse. This means that the area of the region is half of the area of the ellipse. For the area of the whole ellipse, referring to the formula, $\frac{2\pi i}{\sqrt{b^2-4ac}}$ is the area when $f = 1$. Thus, when $f \neq 1$, the formula becomes $\frac{2\pi i}{\frac{1}{f}\sqrt{b^2-4ac}}$ since a, b, c will be scaled down a factor of f. Thus, $\frac{2\pi i}{\frac{1}{f}\sqrt{b^2-4ac}} = \frac{2\pi i}{\frac{1}{24}\sqrt{1-4}} = 16\pi\sqrt{3}$. Half that is $8\pi\sqrt{3}$,

22. A Replace θ with $\theta - \frac{\pi}{4}$ to get $r = \frac{10}{3-2\cos\left(\theta-\frac{\pi}{4}\right)} = \frac{10}{3-\sqrt{2}\cos(\theta)-\sqrt{2}\sin(\theta)} \rightarrow 3r = 10 + \sqrt{2}x + \sqrt{2}y \rightarrow 9r^2 = 9x^2 + 9y^2 = \left(10 + \sqrt{2}x + \sqrt{2}y\right)^2 = 100 + 20\sqrt{2}x + 2x^2 + 20\sqrt{2}y + 4xy + 2y^2 \rightarrow -100 - 20\sqrt{2}x + 7x^2 - 20\sqrt{2}y - 4xy + 7y^2 = 0$

23. B The equation is in the form of $r = \frac{a(1-e^2)}{1-e\cos(\theta)} \rightarrow \frac{10\left(1-\frac{81}{100}\right)}{1-\frac{9}{10}\cos(\theta)} = \frac{19}{10-9\cos\theta}$, which is equivalent to B. Cosine for vertical directrix and - for left of focus.

24. D Obviously, this is an ellipse. We know that $a = 7, c^2 = a^2 - b^2, Area = ab\pi$. We need to minimize c to maximize b. From the information we are given, we know that $14 = \sqrt{(4-1)^2 + (3+1)^2} + \sqrt{(p-1)^2 + (q+1)^2} \rightarrow 81 = (p-1)^2 + (q+1)^2$. This means the other focus lies on the circle $(x-1)^2 + (y+1)^2 = 9^2$. Thus, to minimize the focal length c, the other point must be collinear to $(1,-1)$ and $(4,3)$. Thus, we see that the distance from $(4,3)$ to $(1,-1)$ is $5, (p,q)$ to $(1,-1)$ is $9,$ so $2c = 4 \rightarrow c = 2 \rightarrow b = 3\sqrt{5}$ giving area of $21\pi\sqrt{5}$.

25. A Instead of trying to factor the equation, we can work this problem backwards. We know that $(mx + b - y)(nx + c - y) = bc + bnx - by + cmx - cy + mnx^2 - mxy - nxy + y^2 = 3 + 11x + 10x^2 - 4y - 7xy + y^2$ to keep the coefficients of y the same. This gives us the system of equations: $-b - c = -4, bn + cm = 11, mn = 10, -m - n = -7$. Solving the last two equations gives us $m = 5, n = 2$ or vice versa, it doesn't matter, just keep it consistent when solving for *b, c.* Solving the first two equations gives us $b = 3, c = 1$. So we have $y = 5x + 3$ and $y = 2x + 1$.

26. C The line through P, Q is perpendicular to m which is expressed algebraically by the equation $b(x' - x_1) = a(y' - y)$. Also, $\left(\frac{x_1+x'}{2}, \frac{y_1+y'}{2}\right)$ is the midpoint of $\overline{PQ}$ and is on line m. This geometric fact is expressed algebraically by the equation $a\left(\frac{x_1+x'}{2}\right) + b\left(\frac{y_1+y'}{2}\right) + c = 0$. Rewriting these two previous equations gives us $\begin{cases} bx' - ay' = bx_1 - ay_1 \\ ax' + by' = -2c - ax_1 - by_1 \end{cases} \rightarrow \begin{cases} b^2x' - aby' = b^2x_1 - aby_1 \\ a^2x' + aby' = -2ac - a^2x_1 - aby_1 \end{cases}$. Solving this gives us $x' = \frac{b^2x_1 - a^2x_1 - 2aby_1 - 2ac}{a^2+b^2} = x_1 - \frac{2a(ax_1+by_1+c)}{a^2+b^2}$. Solving for y' in a similar fashion gives us $y' = y_1 - \frac{2b(ax_1+by_1+c)}{a^2+b^2}$.

27. D Using the formulas we just derived, we get $x' = x - \frac{2(x+y-1)}{2} = 1 - y, y' = y - \frac{-2(x+y-1)}{2} = 1 - x$. Plugging this into $-2x' + x'^2 - x'y' + y' + 2y'^2 = 4$ gives us $-3 - 4x + 2x^2 + y - xy + y^2 = 0 \rightarrow -\frac{3}{2} - 2x + x^2 + \frac{y}{2} - \frac{xy}{2} + \frac{y^2}{2} = 0$. The sum is -3.

28. B Let's start at the standard parabola equation $y^2 = 4ax$, where $4a$ is the latus rectum. From there consider the mutually perpendicular chords to have lengths p, q respectively. Hence, the points on the parabola can be written as $(p\cos(\theta), p\sin(\theta))$ and $(q\sin(\theta), q\cos(\theta))$. Plug these points into the parabola equation to see that $p = \frac{4a\cos\theta}{\sin^2\theta}, q = \frac{4a\sin\theta}{\cos^2\theta}$. Now we will eliminate θ.

$$p^{\frac{2}{3}} + q^{\frac{2}{3}} = (4a)^{\frac{2}{3}}\left(\frac{\cos^2\theta + \sin^2\theta}{\sin^{\frac{4}{3}}\theta\cos^{\frac{4}{3}}\theta}\right) = (4a)^{\frac{2}{3}}\left(\frac{1}{\sin\theta\cos\theta}\right)^{\frac{4}{3}} = \frac{(4a)^{\frac{2}{3}}}{(4a)^{\frac{8}{3}}}(pq)^{\frac{4}{3}} \rightarrow (4a)^2 = \frac{(pq)^{\frac{4}{3}}}{p^{\frac{2}{3}}+q^{\frac{2}{3}}} \rightarrow$$

$4a = \sqrt{\frac{(pq)^{\frac{4}{3}}}{p^{\frac{2}{3}}+q^{\frac{2}{3}}}}$. Plugging in values for p and q gives a number close to $\frac{144}{11}$.

29. E 2018 is the radius. So $2018^2\pi$

30. A $x^2 + y^2 + 2x = 0 \rightarrow (x+1)^2 + 2018y^2 = 0 \rightarrow$ the point $(-1,0)$. Answer is 1